

Quark confinement potential examined by excitation energy of the Λ_c and Λ_b baryons in a quark-diquark model

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The possibility to have diquark configuration in heavy baryons, such as Λ_c and Λ_b , is examined by a nonrelativistic potential model with a heavy quark and a light scalar diquark. Assuming that the Λ_c and Λ_b baryons are composed of the heavy quark and the point-like scalar-isoscalar ud diquark, we solve the two-body Schrödinger equation with the Coulomb plus linear potential and obtain the energy spectra for the heavy baryons. Contrary to our expectation, it is found that the potential determined by the quarkonium spectra fails to reproduce the excitation spectra of the Λ_c and Λ_b in the quark-diquark picture, while the Λ_c and Λ_b spectra is reproduced with a half strength of the confinement string tension than for the quarkonium. The finite size effect of the diquark is also examined and it is found that introduction of a finite size diquark would resolve the failure of the spectrum reproduction. The Ξ_c excitation energy is also calculated and is found to be smaller than Λ_c in the quark-diquark model. This is not consistent with the experimental observation.

1. Introduction

Hunts for fundamental correlations in strongly interacting quarks and gluons inside hadron can be a hint to understand the hadron structure and the confinement mechanism. The constituent quarks are considered to be good effective degrees of freedom for the hadron structure, especially explaining the nucleon magnetic moment based on the spin flavor symmetry of the light quarks and low lying energy spectra of the heavy quarkonia by quark excitations in the linear-plus-Coulomb confinement potential. The diquark as two quark pair correlation in hadron can be also a strong candidate of hadron constituent [1, 2]. So far, the existence of diquark correlation inside hadrons has been suggested by phenomenological findings in baryon spectroscopy, weak non-leptonic decays, parton distribution functions and fragmentation functions, and especially the scalar diquark with flavor, spin and color antisymmetric configurations, which is so-called good diquark, is expected to have the most attractive correlation [3]. The strong correlation in the scalar diquark is obtained by one-gluon exchange and by instanton induced interactions, and has been found in Lattice calculations [4–7]. The strong correlation can be a remnant of the Pauli-Gürsey symmetry in two-color QCD, in which quark-quark interactions for massless quarks are exactly same in strength as quark-antiquark interactions. In three-color QCD, this symmetry is explicitly broken and the quark-quark correlations get less attractive than the quark-antiquark correlation.

To investigate the nature of the confinement potential, we focus on the excitation spectra of the heavy baryons. So far, many works have been done for baryons spectra in quark-diquark models. In Ref. [8], the mass spectra of p and d excited states of non-strange baryons were investigated in a diquark-quark model based on the $SU(6) \otimes O(3)$ classification [9] in special attentions of the fine structure by spin-orbit interactions. It was found that the detailed confinement mechanism is irrelevant for the fine structure of the intramultiplet splitting. In Ref. [10], the ground states of spin $3/2$ baryons were investigated in a relativistic formulation. In Ref. [11], radial and orbital excitations of baryons were calculated in a diquark-quark model with a confinement potential reproducing meson spectra, and detailed analyses were given for light flavor baryon spectra. Recently, in Ref. [12], the ground state masses of Λ , Λ_c and Λ_b were calculated in a diquark QCD sum rule, in which a scalar diquark is explicitly considered as a fundamental field in operator product expansion, and the calculation successfully reproduced the observed Λ 's masses with a “constituent” diquark mass 0.4 GeV, satisfying the standard criteria for the QCD sum rule to work well.

The system of a heavy quark and a diquark has an advantage to investigate the light diquark properties. As reported in Ref. [13], in meson wavefunctions, quark-antiquark component are dominated and diquark configuration are rather suppressed. This is because the diquark correlation is weaker than the quark-antiquark correlation and, once there exists a antiquark closed to a diquark, the diquark could easily fall apart and form a quark-antiquark pair. In light baryons, diquark configuration could play an important role for the baryon structure, but rearrangement of the diquark would be also important due to the symmetry among the light quarks. In systems of a heavy quark and two light quarks forming a heavy baryon, thanks to asymmetry between the light quark and the heavy quark, a diquark of two light quarks could easily emerge in the baryon.

In this brief note, taking the quark-diquark picture with a point-like diquark, we report on the excitation energy of the Λ_c and Λ_b baryons using the linear-plus-Coulomb confinement potential suitable for the quarkonium spectra. Here we will find a puzzle that the string tension in the confinement potential for quark and diquark systems should be half as strong as that for quark and antiquark systems to reproduce the Λ_c and Λ_b excitation energies. This is against the universality of the confinement force. To understand the overestimation of these excitation energies, we will examine the finite size effect of the diquark, and find that introduction of the diquark size could be one of the solutions of the above puzzle. We will also find that the excitation energy of the Ξ_c baryon should be smaller than that of Λ_c in the quark-diquark picture, but experiments tell us that it is opposite. These puzzles should be solved when one takes the quark-diquark picture for the heavy baryons.

2. Formulation

We describe the heavy baryons composed of one heavy and two light quarks by the diquark model in which the light quarks form the scalar diquark with antisymmetrized flavor, color anti-triplet $\bar{\mathbf{3}}$ and spin-parity 0^+ . We regard the heavy quark (a charm or bottom quark) and the scalar diquark as elementary particles and take a quark potential model in the non-relativistic formulation. We also assume that the diquark is a point-like particle. In the center of mass frame, the Hamiltonian of the system including the rest masses of the quark

and diquark is written as

$$H = m_h + m_d + \frac{p^2}{2\mu} + V(r) \quad (1)$$

with the heavy quark mass m_h , the diquark mass m_d , the momentum of the relative motion p and the reduced mass μ . We use the c and b quark masses as $m_c = 1.5 \text{ GeV}/c^2$ and $m_b = 4.0/c^2$. The interaction between the heavy quark and the scalar diquark is assumed to be spherical and written as a Coulomb plus linear form [14]

$$V(r) = -\frac{4}{3}\frac{\alpha}{r}\hbar c + kr + V_0 \quad (2)$$

with three constant parameters α , k and V_0 . The Coulomb part proportional to r^{-1} expresses the asymptotic nature of the strong interaction in short distance, and the linear part, kr , represents the confinement potential with the string tension k of the flux tube. The constant V_0 adjusts the absolute value of the potential energy, turning out to be an irrelevant parameter in the present analysis because we are interested in the excitation energies of quark-diquark systems. It is known that the potential $V(r)$ given in Eq. (2) reproduces the charmonium and bottonium spectra well with appropriate spin-spin and spin-orbit interactions [15]. The scalar diquark of our interest has an anti-triplet color charge and it is same as the color charge of antiquark. Because the color electric force should be independent of the quark flavor in the first approximation and be determined by the color charges, we may make good use of the potential determined by the quarkonium spectra for the quark-diquark system.

The energy spectrum and the wave function of the quark-diquark system can be obtained by solving the Schödinger equation for the radial variable of the relative motion:

$$-\frac{\hbar^2}{2\mu}\frac{d^2\chi_\ell(r)}{dr^2} + \left[V(r) + \frac{\ell(\ell+1)\hbar^2}{2\mu r^2}\right]\chi_\ell(r) = E_\ell\chi_\ell(r), \quad (3)$$

where μ is the reduced mass of the heavy quark and scalar diquark and $\chi_\ell(r) \equiv R_\ell(r)/r$ is the radial wave function for the orbital angular momentum $\ell\hbar$. The mass of the bound system for the heavy quark and the scalar diquark is given by $M = m_h + m_d + E_\ell$. Because this Schödinger equation is a one-body equation after separating out the center of mass motion, it is simple enough to be solved numerically. We use the state label defined by $n_r\ell$ with the quantum numbers, n_r and ℓ , for the radial excitation and the orbital angular momentum, respectively. We are of great interest in the excitation pattern of the quark-diquark system in the confinement potential. We focus on the energy spectrum without considering the fine structure caused by the spin-orbit interaction. The fine structure is not sensitive to the details of the confinement mechanism [8]. We may refine our model quantitatively, for instance, by fine-tuning quark and diquark masses, or introducing scale dependence of the Coulomb part of the confinement potential. These are important next steps to be done. Here we are of great interest in the global structure of the excitation spectra of the heavy baryon. Thus, as a first step, we ignore these corrections and spin-orbit interaction.

3. Results

In this section, we present our results of the energy spectra obtained by solving the Schrödinger equation (3) with the confinement potential (2). First we show the charmonium and bottonium spectra to confirm that the confinement potential with $\alpha = 0.4$ and

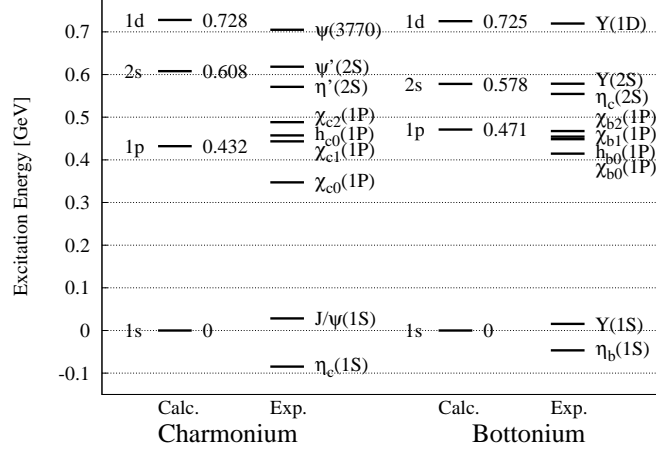


Fig. 1 Excitation spectra of charmonium and bottomonium in our model with $\alpha = 0.4$, $k = 0.9$ GeV/fm, $m_c = 1.5$ GeV/ c^2 and $m_b = 4.0$ GeV/ c^2 . In the left line for each quarkonium the calculated values are shown with the state label and a number. The numbers are excitation energies in units of GeV measured from the ground state energy. The states are labeled by $n_r\ell$ with the radial quantum number n_r and the angular momentum quantum number ℓ . The calculation is done without spin-spin nor spin-orbit interaction. In the right line for each quarkonium we show the excitation energies of the observed quarkonia measured from the spin weighted average of the $1s$ ground state energies. The experimental data are taken from Particle Data Group [17].

$k = 0.9$ GeV/fm reproduces the observed major excitation well. We will also see that with the same parameters the model works for the heavy mesons, D , D_s , B , B_s , with the light quark mass $m_q = 0.3$ GeV/ c^2 and the strange quark mass $m_s = 0.5$ GeV/ c^2 . Next, finding that the confinement potential suitable for the meson spectra does *not* work in the quark-diquark systems to reproduce the excitation energies of Λ_c and Λ_b , we search for the appropriate parameter set to reproduce the Λ_c and Λ_b spectra. We will calculate the Ξ_c and Ξ_b excitation energies by introducing the strange scalar diquark composed of a strange quark and a light quark. We also estimate the size of the decay width of the excited state of Λ_c based on the quark-diquark picture by applying Gamov theory for the nuclear α decay. Finally, we examine the finite size effect of the diquark.

3.1. Quarkonium spectra

Let us first see that the confinement potential (2) with $\alpha = 0.4$ and $k = 0.9$ GeV/fm works well for the charmonium and bottomonium spectra [16]. In Fig. 1, we plot the excitation energy spectra of the charmonium and bottomonium measured from each ground state. The calculation is done with the potential given in Eq. (2) with $\alpha = 0.4$ and $k = 0.9$ GeV/fm without spin-spin nor spin-orbit interaction. The quark masses are fixed as $m_c = 1.5$ GeV/ c^2 for the charm quark and $m_b = 4.0$ GeV/ c^2 for the bottom quark. To compare the calculation without the spin-spin and spin-orbit interaction, the excitation energies of the observed charmonia is measured from the spin wighted average of the η_c and J/ψ masses, $(m_{\eta_c} + 3m_{J/\psi})/4$, which removes the effect of the spin-spin interaction perturbatively, since the matrix element of the

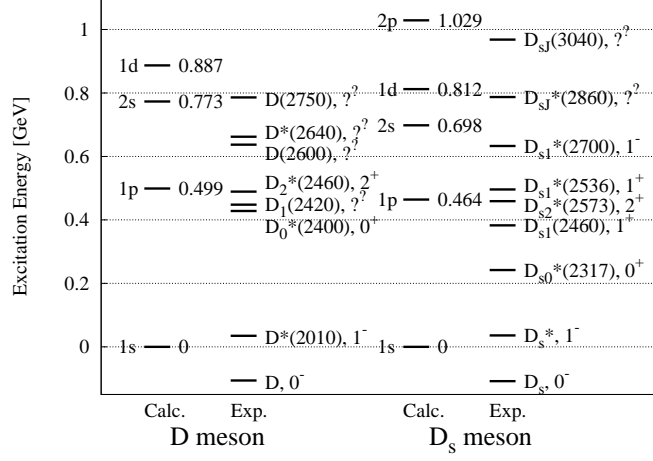


Fig. 2 Same as Fig. 1 for the D and D_s mesons with $m_q = 0.3 \text{ GeV}/c^2$ and $m_s = 0.5 \text{ GeV}/c^2$. For the D meson, we show the spectrum for the charged D meson. The experimental data are taken from Particle Data Group [17].

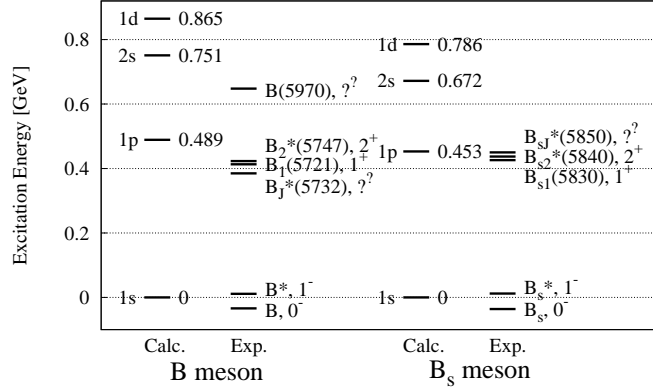


Fig. 3 Same as Fig. 2 for the B and B_s mesons. The experimental data are taken from Particle Data Group [17].

spin-spin operator $\vec{s}_1 \cdot \vec{s}_2$ for the quark-antiquark system with total spin S and the angular momentum $\ell = 0$ is calculated as

$$2\langle S|\vec{s}_1 \cdot \vec{s}_2|S\rangle = S(S+1) - \frac{3}{2} = \begin{cases} -\frac{3}{2} & \text{for } S = 0, \\ +\frac{1}{2} & \text{for } S = 1, \end{cases} \quad (4)$$

In the same way, the excitation energies of the observed bottomia are measured from the spin weighted average of η_b and Υ , $(m_{\eta_b} + 3m_{\Upsilon})/4$. Figure 1 shows that the potential (2) with $\alpha = 0.4$ and $k = 0.9 \text{ GeV}/\text{fm}$ successfully reproduces the size of the major excitations (radial and orbital excitations) both for charmonium and bottomium.

We also calculate the excitation energies of the D and D_s mesons, which are composed of a charm quark and a light (up, down or strange) quark, by using the same potential parameters and the light quark mass $m_q = 0.3 \text{ GeV}/c^2$ and the strange quark mass $m_s = 0.5 \text{ GeV}/c^2$. As shown in Fig. 2, we obtain 0.50 GeV and 0.77 GeV for the D meson excitation energies

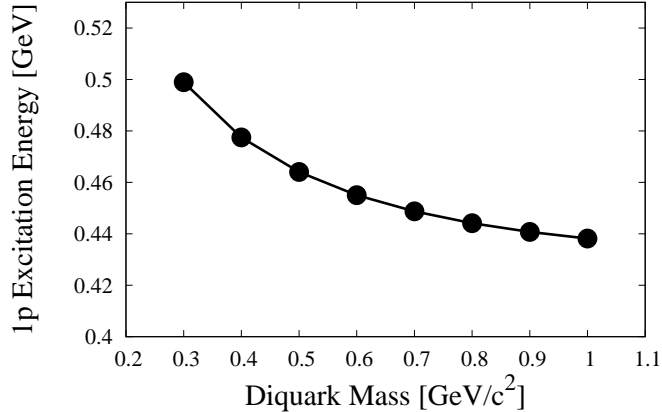


Fig. 4 The Λ_c excitation energy of the first excited state ($1p$) from the ground state ($1s$) as a function of the diquark mass with the c quark mass $m_c = 1.5 \text{ GeV}/c^2$ and the potential parameters $k = 0.9 \text{ GeV/fm}$ and $\alpha = 0.4$, which are suitable for the meson systems. The excitation energy for the observed Λ_c is 0.330 GeV . The quark-diquark model with the confinement potential suitable for the meson systems and a reasonable diquark mass cannot reproduce the observed Λ_c spectrum.

of the $1p$ and $2s$ states of from the ground state ($1s$), respectively. These values should be compared with the experimental data. The experimental excitation energies are shown again from the spin-weighted averaged value of the ground states with $\ell = 0$. Because the excited states of the D and D_s mesons are unstable against the strong interaction with large decay widths and the quantum numbers of some states are not well-known yet, the detailed comparison is somewhat difficult. Nevertheless, it is observed that the excitation energies of the first excited states of the D meson are around 0.45 to 0.57 GeV . These values are very similar with the obtained value by our calculation. Thus, we presume that the excitation of the D meson be also describe by the potential with $\alpha = 0.4$ and $k = 0.9 \text{ GeV/fm}$. In Fig. 3 we show the calculated results for the B and B_s mesons together with the observed spectra. We find that there is no strong discrepancy between the calculation and the observation.

3.2. Mass of diquark in Λ_c

With the success of the potential model for the meson spectra, we would make good use of the same potential parameters for the Λ_c spectrum. Once we have fixed the potential parameters as $\alpha = 0.4$ and $k = 0.9 \text{ GeV/fm}$ in the meson spectra, the diquark mass is only the parameter in the present model. The diquark mass can be fixed, for instance, by the excitation energy of Λ_c . In Fig. 4, we plot the diquark mass dependence of the excitation energy of the first excited state ($1p$) from the ground state ($1s$). The calculation is done without the spin-orbit interaction for the charm quark, which is responsible for the LS splitting between $1/2^-$ and $3/2^-$. In the range of the diquark mass from $0.3 \text{ GeV}/c^2$ to $1.0 \text{ GeV}/c^2$, we obtain the excitation energy in the range of 0.5 GeV to 0.44 GeV . To compare these values with the observed Λ_c excitation energies, we take the spin weighted average of the excitation energies of the $1p$ states, $1/2^-$ and $3/2^-$. The matrix element of the spin-orbit interaction operator is

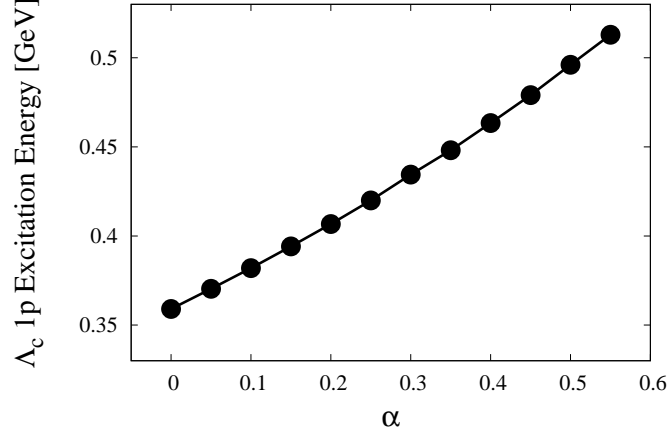


Fig. 5 The Λ_c excitation energy of the first excited state ($1p$) from the ground state ($1s$) as a function of α in the potential (2). The string tension is fixed so as to $k = 0.9$ GeV/fm which is suitable for the meson spectra. The masses of the c quark and the diquark are 1.5 GeV/ c^2 and 0.5 GeV/ c^2 , respectively. The excitation energy for the observed Λ_c is found to be 0.330 GeV.

calculated for the state with total angular momentum J , orbital angular momentum ℓ and spin $s = 1/2$ as

$$2\langle J, \ell, 1/2 | \vec{\ell} \cdot \vec{s} | J, \ell, 1/2 \rangle = J(J+1) - \ell(\ell+1) - \frac{3}{4} = \begin{cases} -2 & \text{for } J = 1/2, \ell = 1 \\ 1 & \text{for } J = 3/2, \ell = 1 \end{cases} \quad (5)$$

Thus, if we take the weighted sum $E_{\text{ave}} = \frac{2}{3}E_{3/2^-} + \frac{1}{3}E_{1/2^-}$, we can remove the effect of the spin-orbit interaction in perturbation theory.

The averaged value of the observed excitation energies of Λ_c is found to be 0.330 GeV. In the following, we refer to this value as experimental value of the $1p$ excitation energy of Λ_c . The experimental value 0.330 GeV is out of the range of the calculated values with the diquark mass from 0.3 GeV/ c^2 to 1.0 GeV/ c^2 , which is an expected range for the diquark as a bound state of two constituent quarks. Thus, we conclude that the confinement potential suitable for the meson systems cannot produce the Λ_c spectrum in the quark-diquark picture with a reasonable diquark mass. This is a very interesting finding. The confinement force in the color electric interaction may be mainly dependent on the color charge and not on the flavor of the colored object. The present result, however, shows that the confinement force is different in the quark-antiquark and quark-diquark systems in the case of the point-like diquark, although the antiquark and diquark have the same color charge. Therefore, as long as we take the quark-diquark picture, we should explain why the confinement potential is different in these systems.

3.3. Potential parameters for Λ_c and Λ_b

We have found that the confinement potential fitted by the quarkonium spectra does not reproduce the excitation energy of the Λ_c baryon in the quark-diquark picture. In this section, we look for the potential parameters appropriate for the Λ_c and Λ_b baryons. Let us fix the

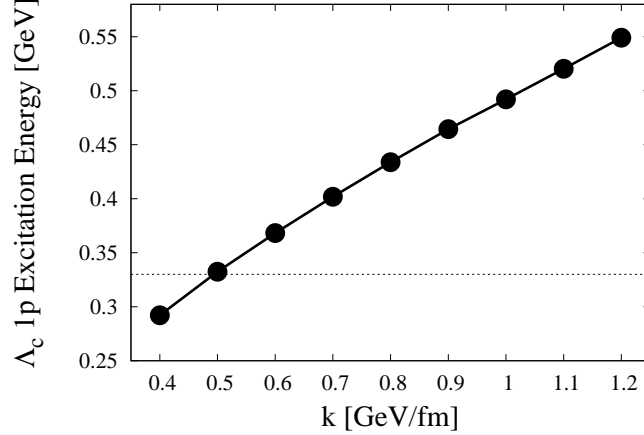


Fig. 6 The Λ_c excitation energy of the first excited state ($1p$) from the ground state ($1s$) as a function of the string tension k in the potential (2). The α parameter is fixed so as to $\alpha = 0.4$. The masses of the c quark and the diquark are $1.5 \text{ GeV}/c^2$ and $0.5 \text{ GeV}/c^2$, respectively. The excitation energy for the observed Λ_c is found to be 0.330 GeV , which is shown as the horizontal dotted line. The string tension around $k = 0.5 \text{ GeV}/\text{fm}$ reproduces the observed excitation energy.

diquark mass as $0.5 \text{ GeV}/c^2$. We have checked that we obtain qualitatively same results also for diquark mass $0.4 \text{ GeV}/c^2$.

First of all, we see the α dependence of the Λ_c excitation energy. In Fig. 5, we plot the excitation energy of the $1p$ state calculated with various values of the α parameter. The string tension k is fixed to be $k = 0.9 \text{ GeV}/\text{fm}$. The figure shows that the experimental excitation energy, 0.330 GeV , cannot be reproduced by α in the range of $\alpha = 0.0$ to 0.6 and $k = 0.9 \text{ GeV}/\text{fm}$.

Next, we look for an appropriate value of the string tension k for the Λ_c spectrum. In Fig. 6, we plot the $1p$ excitation energies calculated with various values of the string tension k . The α parameter is fixed as $\alpha = 0.4$. As we see in the figure, the experimental value, 0.330 GeV , can be reproduced with $k \sim 0.5 \text{ GeV}/\text{fm}$. Calculating also the Λ_b spectrum, we find that a best value of the string tension k for both Λ_c and Λ_b is to be $k = 0.47 \text{ GeV}/\text{fm}$. Figure 7 shows the comparison of the calculated results and observed spectra for Λ_c and Λ_b , suggesting that the potential with $\alpha = 0.4$ and $k = 0.47 \text{ GeV}/\text{fm}$ reproduces the whole excitation spectra of Λ_c and Λ_b . This value is substantially smaller than the string tension obtained for the meson systems, $k = 0.9 \text{ GeV}/\text{fm}$. This implies that the confinement strength of the quark-diquark system is almost half of that in the quark-antiquark system. This is again strange to universality for the confinement potential.

The weaker confinement potential for the quark-diquark system is unlikely to be explained by geometrical argument for interquark flux tube configuration, as long as we take a point-like diquark. As shown in Fig. 8, there are two possibilities of the flux configuration for three-quark baryons, so called Δ -type [18] and Y -type [19, 20]. In the Δ -type configuration, three flux tubes between pairs of quarks form the triangle structure, while in the Y -type configuration there is one junction from which flux tubes are strung to the quarks.

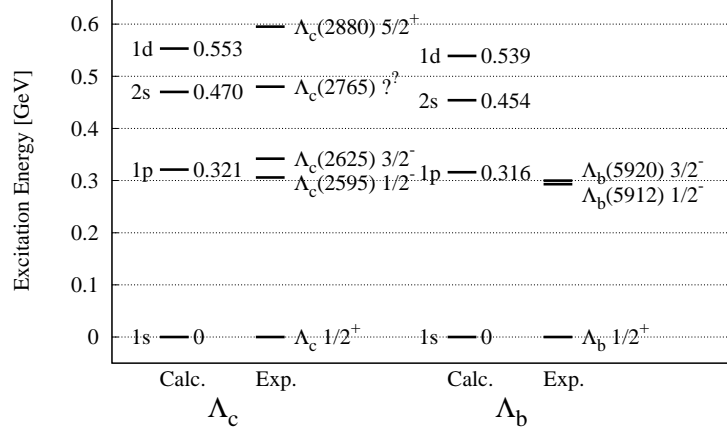


Fig. 7 Comparison of calculated Λ_c and Λ_b excitation energies from the grand state ($1s$) with the experimental data. The potential parameters are fixed as $k = 0.47$ GeV/fm and $\alpha = 0.4$, which reproduce the observed data well. The masses of the c quark, the b quark and the diquark are used as 1.5 GeV/ c^2 , 4.0 GeV/ c^2 and 0.5 GeV/ c^2 , respectively. The experimental data are taken from Particle Data Group [17].

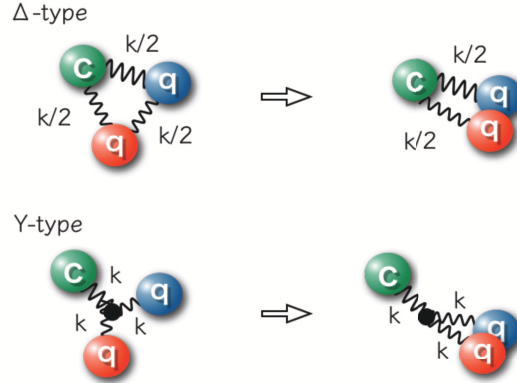


Fig. 8 Schematic figure of interquark flux tube in a baryon and its diquark limit. There are two possibilities of the gluon configuration in a baryon, Δ -type and Y-type.

According to the one-gluon exchange calculation, the attractive interaction between quarks with the $\bar{\mathbf{3}}$ color configuration is half as strong as the interaction between a quark and an antiquark with the color singlet configuration [21, 22]. In the Δ -type configuration, the string tension among the quarks may be given as $k/2$. In this case, if we take the diquark limit where two light quarks come to a same point, the string tension between the heavy quark and diquark may be $k/2 + k/2 = k$ as shown in the upper part of Fig. 8. This does not explain the reduction of the string tension in the quark-diquark system. In the Y-type configuration, the string tension from the junction to the quark has a strength same as the quark-antiquark system with color singlet [20]. If we take the diquark limit, the strength of the string tension between the heavy quark and diquark may be a value between k to $2k$ depending on the position of the junction. For the junction at the diquark, the strength can be k , while the

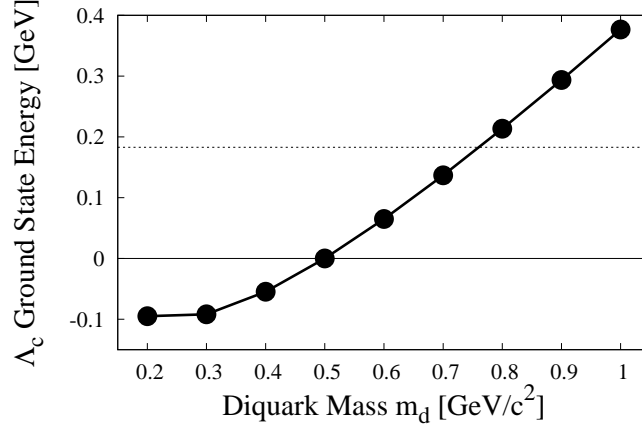


Fig. 9 Diquark mass dependence of the ground state energy of Λ_c ($1s$) measured from the Λ_c mass with $m_d = 0.5$ GeV/c^2 . The horizontal dotted line expresses the mass difference between the observed ground states of Λ_c and Ξ_c . This figure suggests that the strange diquark mass $m_{ds} = 0.76$ GeV/c^2 reproduces the observed Λ_c and Ξ_c mass difference.

strength can be $k + k = 2k$ if the junction located at the heavy quark. This does not again explain the weakness of the string tension for the quark-diquark system.

3.4. Strange diquark mass

Here we consider the scalar strange diquark which is composed of a light quark (up or down) and a strange quark forming the flavor and color antisymmetric configuration ($\bar{\mathbf{3}}$ both for the flavor and color spaces). The strange diquark can be a constituent of Ξ_c (Ξ_b) together with a charm (bottom) quark. Assuming the light flavor symmetry for the confinement potential (2), in which the potential is shared both for the ud diquark and the strange diquark up to the constant part of Eq. (2), V_0 , we can determine the strange diquark mass from, for instance, the mass difference between Λ_c and Ξ_c . In Fig. 9, we show the diquark mass dependence of the charmed baryon mass composed of a charm quark and a scalar diquark. The charmed baryon mass is measured from the Λ_c mass calculated with the diquark mass $m_d = 0.5$ GeV/c^2 , where the Λ_c spectrum is reproduced well. The experimental data for the difference between the Λ_c mass and the isospin averaged Ξ_c mass is 0.18 GeV/c^2 . This corresponds to the calculation with the strange diquark mass $m_{ds} = 0.76$ GeV/c^2 , which is a reasonable value for the strange diquark mass.

With the strange diquark mass $m_{ds} = 0.75$ GeV/c^2 , we calculate the ground state Ξ_b mass. We find that the mass difference between the ground states of Λ_b and Ξ_b is calculated as 0.165 GeV/c^2 . We should compare this value with the experimental mass difference 0.175 GeV/c^2 and find that this is reasonably good agreement. For the ground state Ξ_b mass, we take the isospin average.

3.5. Excitation energy of Ξ_c

With the strange diquark mass $m_{ds} = 0.76$ GeV/c^2 , we calculate the excitation energy of the Ξ_c $1p$ state from its $1s$ ground state. As shown in Fig. 10, the excitation energy is obtained as

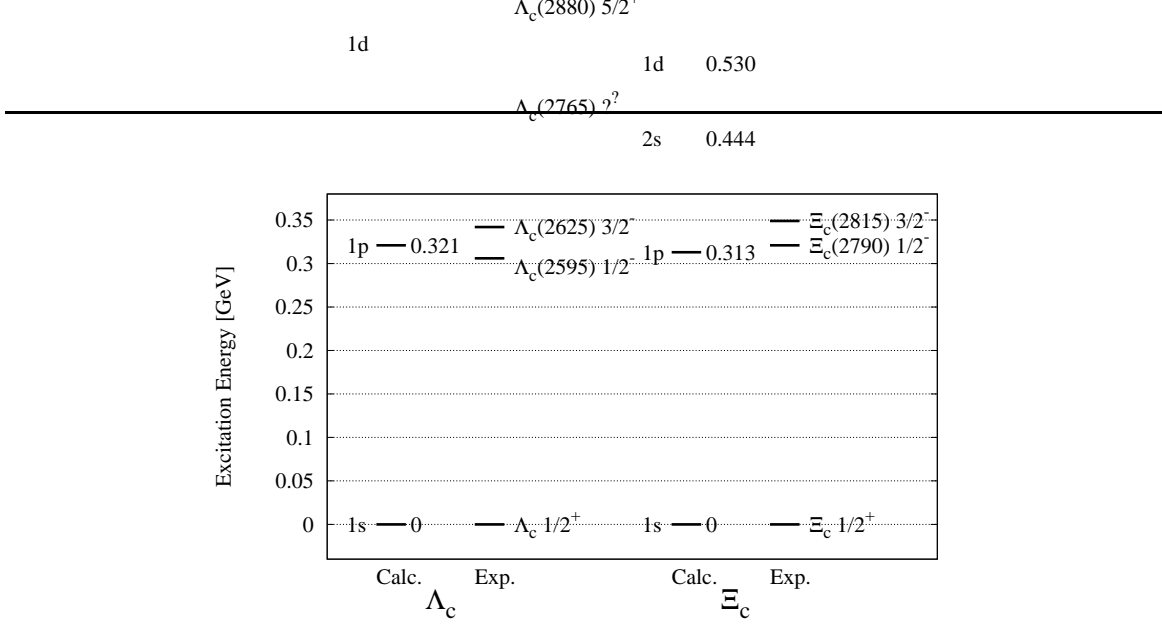


Fig. 10 Excitation energy of the Ξ_c $1p$ state in comparison with Λ_c and the observed excitation energies. The potential parameters are fixed as $k = 0.47$ GeV/fm and $\alpha = 0.4$, which reproduce the observed Λ_c data well. The masses of the c quark and the strange diquark are used as $m_c = 1.5$ GeV/ c^2 and $m_{ds} = 0.76$ GeV/ c^2 , respectively. The experimental data are taken from Particle Data Group [17].

0.313 GeV. Comparing the experimental data shown in the figure, we find that the calculated value underestimates the observed masses of the $1p$ states. The size of the deviation from the observation is as small as the accuracy of the present model, and we do not intend to discuss this discrepancy further quantitatively. Nevertheless, what puzzles us is that the calculated excitation energy of Ξ_c is smaller than that of Λ_c , while the experimental observation tells that they are reversed in order. As shown in Fig. 11, in this potential model the $1p$ Λ_c state with the larger diquark mass has the smaller excitation energy, which fits our intuition that the heavier object is harder to be excited. Unanticipatedly, in nature the excitation energy of Ξ_c is larger than that of Λ_c , although Ξ_c has a heavier component than Λ_c . This is opposite to our model and intuition. Thus, the present result is qualitatively against the observation. This disagreement could be solved by introducing the mixing with the Ξ'_c state in which two light quarks are symmetric in the flavor space. Nevertheless, the Ξ'_c state is a higher excited state than Ξ_c . Usually once the mixing of two states is introduced, level repulsion would take place and the lower state could be pushed down. This is an opposite direction to explain the smaller excitation energy of Ξ_c than Λ_c .

3.6. Decay width of Λ_c excited state

We play one more game with Λ_c in the quark-diquark model with a point-like diquark. We estimate the decay width of the $1p$ excited state of Λ_c . The decay widths of the observed Λ_c states are as narrow as 2.6 MeV for $\Lambda(2595)$ with $1/2^-$ and less than 0.97 MeV for $\Lambda(2625)$ with $3/2^-$, which are extremely small in the strong decay. The main decay mode for both Λ_c is $\pi\pi\Lambda_c$, and $\pi\Lambda_c$ is forbidden by isospin symmetry.

In the flux tube model, pair creation of a quark and an antiquark induces the decay of a hadron into two lighter hadrons. In the quark-diquark model for Λ_c , by the creation of a quark and an antiquark inside of the flux tube, the Λ_c baryon decays into a D meson and a

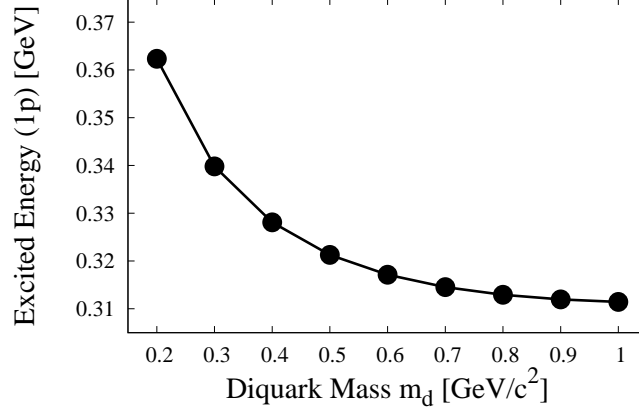


Fig. 11 Diquark mass dependence of the excitation energy of the Λ_c $1p$ state from the $1s$ ground state. This figure shows that we obtain the smaller excitation energy with the larger diquark mass.

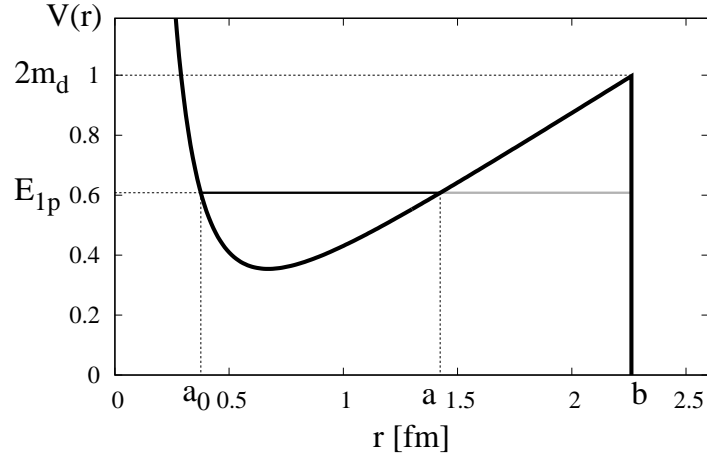


Fig. 12 Potential (6) with the centrifugal potential for $\ell = 1$ as a function of r . The energy E_{1p} is the mass of Λ_c in the $1p$ state measured from $m_c + m_d$. The classical turning points are located at $a_0 = 0.38$ fm and $a = 1.42$ fm. The end of the tunnel is calculated as $b = 2.27$ fm.

nucleon, where the c quark forms the D meson with the created antiquark and the diquark forms the nucleon with the created quark. But, the DN system has a larger rest mass than the Λ_c excited state, so that it cannot decay into DN . With the diquark-antidiquark pair production, the Λ_c excited state decays into Λ_c made of the charm quark and the created diquark and a tetraquark formed by the created antiquark and the diquark originally in the excited Λ_c , and the tetraquark may be a sigma meson and decays into two pions.

Here let us estimate the decay width of Λ_c in the $1p$ state into $\Lambda_c \pi \pi$ by considering pair creation of diquark and antidiquark in the flux tube. We model this situation by taking the

potential to be

$$V(r) = \begin{cases} -\frac{4}{3}\frac{\alpha}{r}\hbar c + kr + V_0 & \text{for } r < b \\ 0 & \text{for } r > b \end{cases} \quad (6)$$

where b is defined by the position where the potential energy reaches the energy for the diquark-antidiquark pair creation, $V(b) = 2m_d$. For larger distance than b , thanks to the pair creation of diquark and antidiquark, there are no interaction and the potential becomes zero. The constant V_0 is determined so as to reproduce the absolute value of the Λ_c ground state mass by the potential (2) together with the center of mass energy $m_c + m_d$. In the present case $V_0 = -0.039$ GeV. In Fig. 12, we plot the potential (6) together with the centrifugal potential for $\ell = 1$. The energy E_{1p} is the mass of Λ_c with $\ell = 1$ measured from $m_c + m_d$. The positions a_0 and a are the classical turning points. The excited Λ_c state with E_{1p} decays by penetrating through the potential barrier from $r = a$ to $r = b$. We calculate the decay width based on Gamov theory for nuclear α decay. There the tunneling probability is calculated by the WKB approximation:

$$P = \exp \left[-\frac{2}{\hbar c} \int_a^b \sqrt{2\mu c^2 (V(r) - E_{1p})} dr \right] = 0.048. \quad (7)$$

For the confined particle in the classical orbit, the numbers of the trials of the penetration per unit time may be calculated as $v_0/(2R)$ with the length of the classical orbit $R = a - a_0$ and the velocity of the particle v_0 . We calculate the velocity by $v_0 = \sqrt{\langle E_{\text{kin}} \rangle / (2\mu)}$ with the expectation value of the kinetic energy $\langle E_{\text{kin}} \rangle$ evaluated with the obtained wavefunction, $\langle E_{\text{kin}} \rangle = 0.125$ GeV. Finally the decay width is estimated as

$$\Gamma = \frac{\hbar}{\tau} = \frac{\hbar c}{R} \sqrt{\frac{\langle E_{\text{kin}} \rangle}{2\mu c^2}} P = 3.8 \text{ [MeV]}, \quad (8)$$

The size of the obtained value is consistent with the observed decay widths of the $1p$ states. It is noted that, since the initial state with $\ell = 1$ has negative parity, the relative angular momentum between Λ_c and the $\pi\pi$ system in the final state should be a odd number, such as $\ell = 1$, for $\pi\pi$ coming from a sigma meson. Nevertheless, in this model we do not consider spin and parity of the decay reaction. Thus, we could have a further suppression factor for parity conserved configurations.

A similar calculation is also done using the potential with $k = 0.9$ GeV/fm. In the calculation we set the mass of the $1p$ state to be the observed mass, which is not an eigenenergy of the potential with $k = 0.9$ GeV/fm. We find the decay width $\Gamma = 31$ MeV, which is one order of magnitude larger than the observed width. This fact also supports the smaller string tension for the heavy quark and diquark system.

3.7. Estimation of the diquark size effect

As reported in Refs. [7, 23], the size of the diquark can be as large as 1 fm. Here we estimate the effect of the diquark size to the excitation spectrum of Λ_c . Let us set the positions of the up, down and charm quarks to be \mathbf{x}_1 , \mathbf{x}_2 and \mathbf{x}_3 , respectively. Assuming that the interaction strength of the inter quark force is half of the force between quark and antiquark, we may

write down the color electric potential between the charm quark and the light quarks as

$$V_{\text{fs}} = -\frac{4}{3}\hbar c \frac{\alpha}{2} \left(\frac{1}{|\mathbf{x}_3 - \mathbf{x}_1|} + \frac{1}{|\mathbf{x}_3 - \mathbf{x}_2|} \right) + \frac{k}{2} (|\mathbf{x}_3 - \mathbf{x}_1| + |\mathbf{x}_3 - \mathbf{x}_2|) \quad (9)$$

where α and k are the coupling strengths appearing in the quark-antiquark potential. Introducing a Jacobi coordinate defined by $\boldsymbol{\rho} = \mathbf{x}_1 - \mathbf{x}_2$ and $\mathbf{r} = \mathbf{x}_3 - \frac{1}{2}(\mathbf{x}_1 + \mathbf{x}_2)$, we write the potential as

$$V_{\text{fs}} = -\frac{4}{3}\hbar c \frac{\alpha}{2} \left(\frac{1}{|\mathbf{r} - \frac{1}{2}\boldsymbol{\rho}|} + \frac{1}{|\mathbf{r} + \frac{1}{2}\boldsymbol{\rho}|} \right) + \frac{k}{2} \left(|\mathbf{r} - \frac{1}{2}\boldsymbol{\rho}| + |\mathbf{r} + \frac{1}{2}\boldsymbol{\rho}| \right) \quad (10)$$

We recover the potential (2) in the limit of $\mathbf{x}_1 = \mathbf{x}_2$, that is $\boldsymbol{\rho} = 0$. Assuming that two light quarks form a diquark, we do not regard $|\boldsymbol{\rho}|$ as a dynamical valuable rather as a parameter for the diquark size. The modulus of \mathbf{r} , $r = |\mathbf{r}|$, represents the distance between the diquark and the charm quark.

Now we try two ways of the estimation of the diquark size effect. First we use perturbation theory by regarding the diquark size to be small. Let us expand the linear potential in terms of the diquark size ρ and take the leading correction:

$$\Delta V_{\text{fs}}^{(1)} = kr \frac{1 - \cos^2 \theta}{8} \frac{\rho^2}{r^2}, \quad (11)$$

where θ is the angle of $\boldsymbol{\rho}$ and \mathbf{r} . We calculate the finite size effect for the Λ_c excitation energy between the 1S and 1P states as first order perturbation:

$$\Delta E = \langle 1P | \Delta V_{\text{fs}}^{(1)} | 1P \rangle - \langle 1S | \Delta V_{\text{fs}}^{(1)} | 1S \rangle. \quad (12)$$

To calculate the matrix elements, we use the wave functions obtained by solving the Schrödinger equation for the unperturbed Hamiltonian with the original parameters for the quarkonia, $\alpha = 0.4$ and $k = 0.9$ GeV/fm, and we assume that $\boldsymbol{\rho}$ orients to the z direction and take $\ell_z = 0$ for the 1P state. Then we find the finite size correction calculated by perturbation theory to be

$$\Delta E = - \left(0.13 \text{ GeV/fm}^2 \right) \rho^2. \quad (13)$$

This shows that the finite size effect reduces the excitation energy, and the magnitude of the correction is to be order of 0.1 GeV for the diquark size 1 fm, although 1 fm should not be a small enough value for perturbative calculation but we could learn the order of magnitude of the size effect. In this calculation the angle between $\boldsymbol{\rho}$ and \mathbf{r} is a dynamical valuable and we do not fix the orientation of the diquark against the charm quark direction.

In the second estimation, we fix the angle between $\boldsymbol{\rho}$ and \mathbf{r} to be orthogonal, but we solve the Schrödinger equation fully. With this assumption the potential is written as

$$V_{\text{fs}}^{(2)}(r) = -\frac{4}{3}\hbar c \frac{\alpha}{\sqrt{r^2 + \frac{1}{4}\rho^2}} + k \sqrt{r^2 + \frac{1}{4}\rho^2} \quad (14)$$

where ρ is the parameter for the diquark size and this potential is a function of the distance between the charm quark and diquark, r . It can be understood from this potential that, for $r < \rho$, the charm quark is located between two light quarks and, consequently, color forces between the charm quark and the light quarks have opposite directions and the forces are to be neutralized. Thus, around the center the color electric force between the charm quark

and the diquark is less attractive than the case of the point-like diquark. For $r > \rho$, ρ would be negligible and we recover the original potential for the point-like diquark. Solving the Schrödinger equation with $\alpha = 0.4$ and $k = 0.9$ GeV/fm, we find the Λ_c excitation energy to be 0.341 GeV for $\rho = 0.5$ fm and 0.273 GeV for $\rho = 1.0$ fm, which should be compared with 0.464 for $\rho = 0.0$ fm. Again we find that the finite size correction reduces the excitation energy and the diquark size 0.5 fm reproduces a reasonable value for the excitation energy, which is consistent with the finding for the finite size diquark in Refs. [7, 23]. The reason of the suppression of the excitation energy for the finite size diquark is the following; As discussed above, for $r < \rho$ the attraction between the charm quark and the diquark gets weaker, and in such a case the distribution of the Λ_c wave function spread out more than the case of the point-like diquark. This makes the centrifugal force less effective. Thus, the orbital excitation energy is reduced for the finite size diquark. For final conclusions, we should investigate the Λ_c spectrum more precisely, for instance, by considering orbital and radial excited states more and the Λ_b spectrum. For quantitatively more precise argument, we should perform three-body calculation by regarding three quarks as dynamical objects, which is, however, beyond the scope of this brief report. Here we just mention that the finite size effect can be a solution of the overestimation problem for the excitation energies, and that the diquark should have color polarization.

4. Summary and conclusion

We have examined the excitation energy of the Λ_c and Λ_b baryons in the quark-diquark model. In this model, Λ_c and Λ_b composed of a heavy quark and a scalar ud diquark with the $\bar{\mathbf{3}}$ configuration for the flavor and color spaces, and these constituents are confined by the Coulomb and linear potential.

We have found that the confinement potential suitable for the meson system overestimates the Λ_c and Λ_b excitation energies, if we assume a point-like diquark. We also find that, in order to reproduce the Λ_c and Λ_b spectra, the string tension should be half as strong as in the meson case. This implies that if one takes the quark-diquark model for the Λ_c and Λ_b baryons with a point-like diquark, one should take such a smaller string tension for the quark-diquark system. This would give us an interesting question why the quark-diquark system should have such a smaller string tension, because the color electric confinement potential is considered to be universal for the same color configuration. We have estimated the decay width of the Λ_c excited state based on Gamov theory of the nuclear alpha decay using the WKB approximation. We have found that the result obtained in this model is consistent with the experiment and this supports the smaller string tension for the quark-diquark model.

The overestimation of the Λ_c excitation energy with the confinement potential suitable for the meson spectra could be resolved by introducing the finite size for the diquark. In this case, for short distance of the heavy quark and the diquark, the color forces between the heavy quark and the light quarks in the diquark have opposite directions and are to be canceled. Thus the interaction between the heavy quark and the diquark for short distance is less attractive. This makes the relative distance of the quark and diquark larger in the Λ_c wave function, and the centrifugal force gets less effective. We have estimated the finite size effect to the Λ_c excitation energy in two ways. One is to use perturbation theory, and the other is to fix the diquark orientation. Both calculations can be done within a simple two-body model. With these calculation, we have found that the finite size effect reduces

the excitation energy and the reduction magnitude would be 0.1 GeV to 0.2 GeV for the diquark size 1 fm. This is a reasonable value to solve the overestimation. Thus, this finding can be one of the confirmations of the finite size diquark in heavy baryons and implies that the diquark should have color polarization. For more precise argument for the diquark size, we certainly need full three body calculation.

We have also calculated the excitation energy of Ξ_c as a system of a charm quark and a strange diquark. We have found that the excitation energy of Ξ_c is moderately smaller than that of Λ_c . This is consistent with our intuition that a heavier particle is harder to be excited, but it is inconsistent with the experimental observation in which the Ξ_c excitation energy is slightly larger than that of Λ_c .

We hope that these findings give us a good chance to reconsider the nature of the confinement force in baryon for excitation spectra of heavy baryons and the feature of diquark in heavy baryons.

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References

- [1] M. Ida and R. Kobayashi, Prog. Theor. Phys. **36**, 846 (1966).
- [2] D. B. Lichtenberg and L. J. Tassie, Phys. Rev. **155**, 1601 (1967).
- [3] R. L. Jaffe, Phys. Rept. **409**, 1 (2005) [Nucl. Phys. Proc. Suppl. **142**, 343 (2005)]
- [4] M. Hess, F. Karsch, E. Laermann and I. Wetzorke, Phys. Rev. D **58**, 111502 (1998).
- [5] R. Babich *et al.* J. High Energy Phys. **01** (2006) 086.
- [6] K. Orginos, Proc. Sci., LAT2005 (2005)054.
- [7] C. Alexandrou, P. de Forcrand and B. Lucini, Phys. Rev. Lett. **97**, 222002 (2006).
- [8] G.R. Goldstein and J. Maharana, Nuovo Cim. **A59**, 393 (1980).
- [9] D.B. Lichtenberg, Phys. Rev. **178**, 2197 (1969).
- [10] D.B. Lichtenberg, W. Namgung, E. Predazzi, and J.G. Wills, Phys. Rev. Lett. **48**, 1653 (1982).
- [11] K.F. Liu and C.W. Wong, Phys. Rev. **D28**, 170 (1983).
- [12] K. Kim, D. Jido, and S.H. Lee, Phys. Rev. C **84**, 025204 (2011).
- [13] R. L. Jaffe, Phys. Rev. D **15**, 267 (1977); 281 (1977).
- [14] E. Eichten, K. Gottfried, T. Kinoshita, J. B. Kogut, K. D. Lane and T. M. Yan, Phys. Rev. Lett. **34**, 369 (1975), Erratum: [Phys. Rev. Lett. **36**, 1276 (1976)].
- [15] S.N. Mukherjee, R. Nag, S. Sanyal, T. Morii, J. Morishita, and M. Tsuge, Phys. Rept. **231**, 201(1993).
- [16] C. Quigg and J. L. Rosner, Phys. Rept. **56**, 167 (1979).
- [17] K.A. Olive *et al.* (Particle Data Group), Chin. Phys. C, **38**, 090001 (2014) and 2015 update.
- [18] J. M. Cornwall, Phys. Rev. D **54**, 6527 (1996).
- [19] S. Capstick and N. Isgur, Phys. Rev. **D34**, 2809 (1986).
- [20] T. T. Takahashi, H. Matsufuru, Y. Nemoto and H. Suganuma, Phys. Rev. Lett. **86**, 18 (2001).
- [21] W. Lucha, F. F. Schoberl and D. Gromes, Phys. Rept. **200**, 127 (1991).
- [22] A. Faessler, F. Fernandez, G. Lubeck and K. Shimizu, Phys. Lett. B **112**, 201 (1982). doi:10.1016/0370-2693(82)90961-3
- [23] S. Imai and H. Suganuma, J. Mod. Phys. **7**, 790 (2016).